A Lock Calculus for Multimode Type Theory (MTT)

Andreas Nuyts

KU Leuven, Belgium

TYPES 2023
Valencia, Spain
June 12, 2023

Crash Course on Multimode Type Theory (MTT)

Let $R : \mathscr{C} \to \mathscr{D}$ be a functor.

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} \mathscr{C}}{R\Gamma\ \mathsf{ctx} \mathbin{@} \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' \mathbin{@} \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' \mathbin{@} \mathscr{D}} \qquad \frac{\Gamma \vdash T\ \mathsf{type} \mathbin{@} \mathscr{C}}{R\Gamma \vdash RT\ \mathsf{type} \mathbin{@} \mathscr{D}} \qquad \frac{\Gamma \vdash t : T \mathbin{@} \mathscr{C}}{R\Gamma \vdash Rt : RT \mathbin{@} \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT\ \mathsf{type}}$$

We check $\Gamma \vdash T\ \mathsf{type} \mathbin{@} \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context Γ such that $\sigma : \Delta \to R\Gamma$ exists.

This is **exactly** what a **left adjoint** gives you.

Let $R : \mathscr{C} \to \mathscr{D}$ be a CwF morphism.

$$\frac{\Gamma\ \mathsf{ctx} @ \mathscr{C}}{R\Gamma\ \mathsf{ctx} @ \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' @ \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' @ \mathscr{D}} \qquad \frac{\Gamma \vdash T\ \mathsf{type} @ \mathscr{C}}{R\Gamma \vdash RT\ \mathsf{type} @ \mathscr{D}} \qquad \frac{\Gamma \vdash t : T @ \mathscr{C}}{R\Gamma \vdash Rt : RT @ \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT\ \mathsf{type}}$$

We check $\Gamma \vdash T\ \mathsf{type} @ \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context Γ such that $\sigma : \Delta \to R\Gamma$ exists.

This is **exactly** what a **left adjoint** gives you.

Let $R : \mathscr{C} \to \mathscr{D}$ be a CwF morphism.

$$\frac{\Gamma\ \mathsf{ctx} \ @\ \mathscr{C}}{R\Gamma\ \mathsf{ctx} \ @\ \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' \ @\ \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' \ @\ \mathscr{D}} \qquad \frac{\Gamma \vdash T\ \mathsf{type} \ @\ \mathscr{C}}{R\Gamma \vdash RT\ \mathsf{type} \ @\ \mathscr{D}} \qquad \frac{\Gamma \vdash t : T \ @\ \mathscr{C}}{R\Gamma \vdash Rt : RT \ @\ \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT\ \mathsf{type}}$$

We check $\Gamma \vdash T\ \mathsf{type} \ @\ \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context Γ such that $\sigma : \Delta \to R\Gamma$ exists.

This is **exactly** what a **left adjoint** gives you.

Let $R : \mathscr{C} \to \mathscr{D}$ be a CwF morphism.

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ \mathscr{C}}{R\Gamma \ \mathsf{ctx} \ @ \ \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' \ @ \ \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' \ @ \ \mathscr{D}} \qquad \frac{\Gamma \vdash T \ \mathsf{type} \ @ \ \mathscr{C}}{R\Gamma \vdash RT \ \mathsf{type} \ @ \ \mathscr{D}} \qquad \frac{\Gamma \vdash t : T \ @ \ \mathscr{C}}{R\Gamma \vdash Rt : RT \ @ \ \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT \ \mathsf{type}}$$

We check $\Gamma \vdash T$ type $@ \ \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context Γ such that $\sigma : \Delta \to R\Gamma$ exists.

This is **exactly** what a **left adjoint** gives you.

Let $R : \mathscr{C} \to \mathscr{D}$ be a CwF morphism.

$$\frac{\Gamma \,\mathsf{ctx} \,@\, \mathscr{C}}{R\Gamma \,\mathsf{ctx} \,@\, \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' \,@\, \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' \,@\, \mathscr{D}} \qquad \frac{\Gamma \vdash T \,\mathsf{type} \,@\, \mathscr{C}}{R\Gamma \vdash RT \,\mathsf{type} \,@\, \mathscr{D}} \qquad \frac{\Gamma \vdash t : T \,@\, \mathscr{C}}{R\Gamma \vdash Rt : RT \,@\, \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT \,\mathsf{type}}$$

We check $\Gamma \vdash T$ type $@\, \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context Γ such that $\sigma : \Delta \to R\Gamma$ exists.

This is **exactly** what a **left adjoint** gives you.

Let $R : \mathscr{C} \to \mathscr{D}$ be a CwF morphism.

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} \mathscr{C}}{R\Gamma\ \mathsf{ctx} \mathbin{@} \mathscr{D}} \qquad \frac{\tau : \Gamma \to \Gamma' \mathbin{@} \mathscr{C}}{R\tau : R\Gamma \to R\Gamma' \mathbin{@} \mathscr{D}} \qquad \frac{\Gamma \vdash T\ \mathsf{type} \mathbin{@} \mathscr{C}}{R\Gamma \vdash RT\ \mathsf{type} \mathbin{@} \mathscr{D}} \qquad \frac{\Gamma \vdash t : T \mathbin{@} \mathscr{C}}{R\Gamma \vdash Rt : RT \mathbin{@} \mathscr{D}}$$

Ok, so how do we check

$$\frac{?}{\Delta \vdash RT\ \mathsf{type}}$$

We check $\Gamma \vdash T\ \mathsf{type} \mathbin{@} \mathscr{C}$ and substitute with $\sigma : \Delta \to R\Gamma$.
BUT: Don't bother the user. Synthesize Γ and σ.

$\Gamma \in \mathscr{C}$ should be the **universal** context $L\Delta$ such that $\eta_\Delta : \Delta \to RL\Delta$ exists.

This is **exactly** what a **left adjoint** gives you.

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma \text{ ctx} @ q}{\Gamma, 🔒_\mu \text{ ctx} @ p} \qquad \frac{\Gamma, 🔒_\mu \vdash T \text{ type} @ p}{\Gamma \vdash \langle \mu \mid T \rangle \text{ type} @ q} \qquad \frac{\Gamma, 🔒_\mu \vdash t : T @ p}{\Gamma \vdash \text{mod}_\mu\, t : \langle \mu \mid T \rangle @ q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![🔒_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![🔑_\alpha]\!] : [\![🔒_\nu]\!] \to [\![🔒_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ q}{\Gamma, \text{🔒}_\mu \ \mathsf{ctx} \ @ \ p} \qquad \frac{\Gamma, \text{🔒}_\mu \vdash T \ \mathsf{type} \ @ \ p}{\Gamma \vdash \langle \mu \mid T \rangle \ \mathsf{type} \ @ \ q} \qquad \frac{\Gamma, \text{🔒}_\mu \vdash t : T \ @ \ p}{\Gamma \vdash \mathsf{mod}_\mu \ t : \langle \mu \mid T \rangle \ @ \ q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![\text{🔒}_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![\text{🔑}_\alpha]\!] : [\![\text{🔒}_\nu]\!] \to [\![\text{🔒}_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} q}{\Gamma, 🔒_\mu\ \mathsf{ctx} \mathbin{@} p} \qquad \frac{\Gamma, 🔒_\mu \vdash T\ \mathsf{type} \mathbin{@} p}{\Gamma \vdash \langle \mu \mid T \rangle\ \mathsf{type} \mathbin{@} q} \qquad \frac{\Gamma, 🔒_\mu \vdash t : T \mathbin{@} p}{\Gamma \vdash \mathsf{mod}_\mu\, t : \langle \mu \mid T \rangle \mathbin{@} q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![🔒_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![🔑_\alpha]\!] : [\![🔒_\nu]\!] \to [\![🔒_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma \,\mathsf{ctx} \,@\, q}{\Gamma, 🔒_\mu \,\mathsf{ctx} \,@\, p} \qquad \frac{\Gamma, 🔒_\mu \vdash T \,\mathsf{type} \,@\, p}{\Gamma \vdash \langle \mu \mid T \rangle \,\mathsf{type} \,@\, q} \qquad \frac{\Gamma, 🔒_\mu \vdash t : T \,@\, p}{\Gamma \vdash \mathsf{mod}_\mu\, t : \langle \mu \mid T \rangle \,@\, q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![🔒_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![🗝_\alpha]\!] : [\![🔒_\nu]\!] \to [\![🔒_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma \text{ ctx} @ q}{\Gamma, \text{🔒}_\mu \text{ ctx} @ p} \qquad \frac{\Gamma, \text{🔒}_\mu \vdash T \text{ type} @ p}{\Gamma \vdash \langle \mu \mid T \rangle \text{ type} @ q} \qquad \frac{\Gamma, \text{🔒}_\mu \vdash t : T @ p}{\Gamma \vdash \text{mod}_\mu\, t : \langle \mu \mid T \rangle @ q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![\text{🔒}_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![\text{🔑}_\alpha]\!] : [\![\text{🔒}_\nu]\!] \to [\![\text{🔒}_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Multimodal TT (MTT)

MTT [GKNB21] is parametrized by a **2-category** (the **mode theory**):

- modes p, q, r, ...
- modalities $\mu : p \to q$

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} q}{\Gamma, 🔒_\mu\ \mathsf{ctx} \mathbin{@} p} \qquad \frac{\Gamma, 🔒_\mu \vdash T\ \mathsf{type} \mathbin{@} p}{\Gamma \vdash \langle \mu \mid T \rangle\ \mathsf{type} \mathbin{@} q} \qquad \frac{\Gamma, 🔒_\mu \vdash t : T \mathbin{@} p}{\Gamma \vdash \mathsf{mod}_\mu\ t : \langle \mu \mid T \rangle \mathbin{@} q}$$

- 2-cells $\alpha : \mu \Rightarrow \nu$.

Semantics:

- $[\![p]\!]$ is a CwF (category with families) modelling all of DTT,
- $[\![\mu]\!]$ is a (weak) dependent right adjoint (DRA) [BCMMPS20] to $[\![🔒_\mu]\!] : [\![q]\!] \to [\![p]\!]$,
- $[\![🔑_\alpha]\!] : [\![🔒_\nu]\!] \to [\![🔒_\mu]\!]$ and corresponding $[\![\alpha]\!] : [\![\mu]\!] \to [\![\nu]\!]$

Plain DTT

$$\frac{}{() \text{ ctx}} \qquad \frac{\Gamma \text{ ctx} \quad \Gamma \vdash T \text{ type}}{\Gamma, x : T \text{ ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p \text{ mode}}{() \text{ ctx} @ p} \qquad \frac{\Gamma \text{ ctx} @ q \quad \mu : p \to q \quad \Gamma, \text{🔒}_\mu \vdash T \text{ type} @ p}{\Gamma, \mu \mid x : T \text{ ctx} @ q}$$

MTT: Locks

$$\frac{\Gamma \text{ ctx} @ q \quad \mu : p \to q}{\Gamma, \text{🔒}_\mu \text{ ctx} @ p}$$

Strict functoriality:

- $\Gamma, \text{🔒}_{\text{id}} = \Gamma,$
- $\Gamma, \text{🔒}_{\nu \circ \mu} = \Gamma, \text{🔒}_\nu, \text{🔒}_\mu$

Contexts in MTT

Plain DTT

$$\frac{}{() \ \mathsf{ctx}} \qquad \frac{\Gamma \ \mathsf{ctx} \quad \Gamma \vdash T \ \mathsf{type}}{\Gamma, x : T \ \mathsf{ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p \ \mathsf{mode}}{() \ \mathsf{ctx} \ @ \ p} \qquad \frac{\Gamma \ \mathsf{ctx} \ @ \ q \quad \mu : p \to q \quad \Gamma, 🔒_\mu \vdash T \ \mathsf{type} \ @ \ p}{\Gamma, \mu \mid x : T \ \mathsf{ctx} \ @ \ q}$$

MTT: Locks

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ q \quad \mu : p \to q}{\Gamma, 🔒_\mu \ \mathsf{ctx} \ @ \ p}$$

Strict functoriality:

- $\Gamma, 🔒_{\mathrm{id}} = \Gamma,$
- $\Gamma, 🔒_{\nu \circ \mu} = \Gamma, 🔒_\nu, 🔒_\mu$

Contexts in MTT

Plain DTT

$$\frac{}{() \ \mathsf{ctx}} \qquad \frac{\Gamma \ \mathsf{ctx} \quad \Gamma \vdash T \ \mathsf{type}}{\Gamma, x : T \ \mathsf{ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p \ \mathsf{mode}}{() \ \mathsf{ctx} \ @ \ p} \qquad \frac{\Gamma \ \mathsf{ctx} \ @ \ q \quad \mu : p \to q \quad \Gamma, \text{🔒}_\mu \vdash T \ \mathsf{type} \ @ \ p}{\Gamma, \mu \mid x : T \ \mathsf{ctx} \ @ \ q}$$

MTT: Locks

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ q \qquad \mu : p \to q}{\Gamma, \text{🔒}_\mu \ \mathsf{ctx} \ @ \ p}$$

Strict functoriality:

- $\Gamma, \text{🔒}_{\mathrm{id}} = \Gamma,$
- $\Gamma, \text{🔒}_{\nu \circ \mu} = \Gamma, \text{🔒}_\nu, \text{🔒}_\mu$

Plain DTT

$$\frac{}{()\ \mathsf{ctx}} \qquad \frac{\Gamma\ \mathsf{ctx} \quad \Gamma \vdash T\ \mathsf{type}}{\Gamma, x : T\ \mathsf{ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p\ \mathsf{mode}}{()\ \mathsf{ctx} @ p} \qquad \frac{\Gamma\ \mathsf{ctx} @ q \quad \mu : p \to q \quad \Gamma, \text{🔒}_{\mu} \vdash T\ \mathsf{type} @ p}{\Gamma, \mu \mid x : T\ \mathsf{ctx} @ q}$$

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q}{\Gamma, \text{🔒}_{\mu}\ \mathsf{ctx} @ p}$$

Strict functoriality:

- $\Gamma, \text{🔒}_{\mathrm{id}} = \Gamma,$
- $\Gamma, \text{🔒}_{\nu \circ \mu} = \Gamma, \text{🔒}_{\nu}, \text{🔒}_{\mu}$

Contexts in MTT

Plain DTT

$$\frac{}{()\ \mathsf{ctx}} \qquad \frac{\Gamma\ \mathsf{ctx} \qquad \Gamma \vdash T\ \mathsf{type}}{\Gamma, x : T\ \mathsf{ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p\ \mathsf{mode}}{()\ \mathsf{ctx} @ p} \qquad \frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q \qquad \Gamma, 🔒_\mu \vdash T\ \mathsf{type} @ p}{\Gamma, \mu \mid x : T\ \mathsf{ctx} @ q}$$

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q}{\Gamma, 🔒_\mu\ \mathsf{ctx} @ p}$$

Strict functoriality:

- $\Gamma, 🔒_{\mathrm{id}} = \Gamma,$
- $\Gamma, 🔒_{\nu \circ \mu} = \Gamma, 🔒_\nu, 🔒_\mu$

Contexts in MTT

Plain DTT

$$\frac{}{()\ \mathsf{ctx}} \qquad \frac{\Gamma\ \mathsf{ctx} \quad \Gamma \vdash T\ \mathsf{type}}{\Gamma, x : T\ \mathsf{ctx}}$$

$\Rightarrow$ Contexts are lists of types.

MTT: Modal variables

$$\frac{p\ \mathsf{mode}}{()\ \mathsf{ctx} @ p} \qquad \frac{\Gamma\ \mathsf{ctx} @ q \quad \mu : p \to q \quad \Gamma, 🔒_\mu \vdash T\ \mathsf{type} @ p}{\Gamma, \mu \mid x : T\ \mathsf{ctx} @ q}$$

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} @ q \quad \mu : p \to q}{\Gamma, 🔒_\mu\ \mathsf{ctx} @ p}$$

Strict functoriality:

- $\Gamma, 🔒_{\mathrm{id}} = \Gamma,$
- $\Gamma, 🔒_{\nu \circ \mu} = \Gamma, 🔒_\nu, 🔒_\mu$

Substitutions in MTT

Plain DTT

$$\frac{\Delta\ \mathsf{ctx}}{() : \Delta \to ()}$$

$$\frac{\sigma : \Delta \to \Gamma \qquad \Delta \vdash t : T[\sigma]}{(\sigma, t/x) : \Delta \to (\Gamma, x : T)}$$

$\Rightarrow$ Substitutions are lists of terms, to be substituted for variables.

MTT: Modal variables

$$\frac{\Delta\ \mathsf{ctx} \ @\ p \qquad p\ \mathsf{mode}}{() : \Delta \to () \ @\ p}$$

$$\frac{\sigma : \Delta \to \Gamma \ @\ q \qquad \mu : p \to q \qquad \Delta, 🔒_\mu \vdash t : T[\sigma, 🔒_\mu] \ @\ p}{(\sigma, t/x) : \Delta \to (\Gamma, \mu \mid x : T) \ @\ q}$$

MTT: Locking is bifunctorial

$$\frac{\sigma : \Delta \to \Gamma \ @\ p \qquad \alpha : \mu \Rightarrow \nu}{(\sigma, 🔑_\alpha) : (\Delta, 🔒_\nu) \to (\Gamma, 🔒_\mu)}$$

So contexts & substitutions are no longer what we're used to.

Substitutions in MTT

Plain DTT

$$\frac{\Delta\ \mathrm{ctx}}{() : \Delta \to ()}$$

$$\frac{\sigma : \Delta \to \Gamma \qquad \Delta \vdash t : T[\sigma]}{(\sigma, t/x) : \Delta \to (\Gamma, x : T)}$$

$\Rightarrow$ Substitutions are lists of terms, to be substituted for variables.

MTT: Modal variables

$$\frac{\Delta\ \mathrm{ctx} \,@\, p \qquad p\ \mathrm{mode}}{() : \Delta \to () \,@\, p}$$

$$\frac{\sigma : \Delta \to \Gamma \,@\, q \qquad \mu : p \to q \qquad \Delta, \text{🔒}_\mu \vdash t : T[\sigma, \text{🔒}_\mu] \,@\, p}{(\sigma, t/x) : \Delta \to (\Gamma, \mu \mid x : T) \,@\, q}$$

MTT: Locking is bifunctorial

$$\frac{\sigma : \Delta \to \Gamma \,@\, p \qquad \alpha : \mu \Rightarrow \nu}{(\sigma, \text{🔑}_\alpha) : (\Delta, \text{🔒}_\nu) \to (\Gamma, \text{🔒}_\mu)}$$

So contexts & substitutions are no longer what we're used to.

Substitutions in MTT

Plain DTT

$$\frac{\Delta\ \mathrm{ctx}}{() : \Delta \to ()}$$

$$\frac{\sigma : \Delta \to \Gamma \qquad \Delta \vdash t : T[\sigma]}{(\sigma, t/x) : \Delta \to (\Gamma, x : T)}$$

$\Rightarrow$ Substitutions are lists of terms, to be substituted for variables.

MTT: Modal variables

$$\frac{\Delta\ \mathrm{ctx} \mathbin{@} p \qquad p\ \mathrm{mode}}{() : \Delta \to () \mathbin{@} p}$$

$$\frac{\sigma : \Delta \to \Gamma \mathbin{@} q \qquad \mu : p \to q \qquad \Delta, \text{🔒}_\mu \vdash t : T[\sigma, \text{🔒}_\mu] \mathbin{@} p}{(\sigma, t/x) : \Delta \to (\Gamma, \mu \mid x : T) \mathbin{@} q}$$

MTT: Locking is bifunctorial

$$\frac{\sigma : \Delta \to \Gamma \mathbin{@} p \qquad \alpha : \mu \Rightarrow \nu}{(\sigma, \text{🔑}_\alpha) : (\Delta, \text{🔒}_\nu) \to (\Gamma, \text{🔒}_\mu)}$$

So contexts & substitutions are no longer what we're used to.

Substitutions in MTT

Plain DTT

$$\frac{\Delta\ \mathsf{ctx}}{() : \Delta \to ()}$$

$$\frac{\begin{array}{l}\sigma : \Delta \to \Gamma \\ \Delta \vdash t : T[\sigma]\end{array}}{(\sigma, t/x) : \Delta \to (\Gamma, x : T)}$$

$\Rightarrow$ Substitutions are lists of terms, to be substituted for variables.

MTT: Modal variables

$$\frac{\Delta\ \mathsf{ctx} \mathbin{@} p \qquad p\ \mathsf{mode}}{() : \Delta \to () \mathbin{@} p}$$

$$\frac{\begin{array}{l}\sigma : \Delta \to \Gamma \mathbin{@} q \qquad \mu : p \to q \\ \Delta, 🔒_\mu \vdash t : T[\sigma, 🔒_\mu] \mathbin{@} p\end{array}}{(\sigma, t/x) : \Delta \to (\Gamma, \mu \mid x : T) \mathbin{@} q}$$

MTT: Locking is bifunctorial

$$\frac{\begin{array}{l}\sigma : \Delta \to \Gamma \mathbin{@} p \\ \alpha : \mu \Rightarrow \nu\end{array}}{(\sigma, 🔑_\alpha) : (\Delta, 🔒_\nu) \to (\Gamma, 🔒_\mu)}$$

So contexts & substitutions are no longer what we're used to.

Substitutions in MTT

Plain DTT

$$\frac{\Delta\,\mathsf{ctx}}{() : \Delta \to ()}$$

$$\frac{\sigma : \Delta \to \Gamma \qquad \Delta \vdash t : T[\sigma]}{(\sigma, t/x) : \Delta \to (\Gamma, x : T)}$$

$\Rightarrow$ Substitutions are lists of terms, to be substituted for variables.

MTT: Modal variables

$$\frac{\Delta\,\mathsf{ctx} \mathbin{@} p \qquad p\,\mathsf{mode}}{() : \Delta \to () \mathbin{@} p}$$

$$\frac{\sigma : \Delta \to \Gamma \mathbin{@} q \qquad \mu : p \to q \qquad \Delta, \text{🔒}_\mu \vdash t : T[\sigma, \text{🔒}_\mu] \mathbin{@} p}{(\sigma, t/x) : \Delta \to (\Gamma, \mu \mid x : T) \mathbin{@} q}$$

MTT: Locking is bifunctorial

$$\frac{\sigma : \Delta \to \Gamma \mathbin{@} p \qquad \alpha : \mu \Rightarrow \nu}{(\sigma, \text{🔑}_\alpha) : (\Delta, \text{🔒}_\nu) \to (\Gamma, \text{🔒}_\mu)}$$

So contexts & substitutions are no longer what we're used to.

The Lock Calculus

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then $🔒_\kappa$ is **pointed** by $🔑_\varepsilon : \mathrm{Id} \to 🔒_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, 🔒_\kappa, 🔒_\kappa, 🔒_\kappa \vdash t : T}{\Gamma, 🔒_\kappa, 🔒_\kappa \vdash t[🔑_\varepsilon] : T[🔑_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔒_\kappa, 🔑_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔑_\varepsilon, 🔒_\kappa)$ or
- $(\mathrm{id}_\Gamma, 🔑_\varepsilon, 🔒_\kappa, 🔒_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} \mathrel{@} q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : 🔒_\mu\ \mathsf{ctx} \mathrel{@} p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx} \mathrel{@} q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : 🔒_\mu \vdash T\ \mathsf{type} \mathrel{@} p}{\Gamma, x : \{\mathfrak{m} : 🔒_\mu\} \rhd T\ \mathsf{ctx} \mathrel{@} q}$$

$$\frac{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{j} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t[🔑_\varepsilon()/\mathfrak{j}] : T[🔑_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then $🔒_\kappa$ is **pointed** by $🔑_\varepsilon : \mathrm{Id} \to 🔒_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, 🔒_\kappa, 🔒_\kappa, 🔒_\kappa \vdash t : T}{\Gamma, 🔒_\kappa, 🔒_\kappa \vdash t[🔑_\varepsilon] : T[🔑_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔒_\kappa, 🔑_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔑_\varepsilon, 🔒_\kappa)$ or
- $(\mathrm{id}_\Gamma, 🔑_\varepsilon, 🔒_\kappa, 🔒_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma \;\mathsf{ctx} \;@\; q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : 🔒_\mu \;\mathsf{ctx} \;@\; p}$$

MTT: Modal variables

$$\frac{\Gamma \;\mathsf{ctx} \;@\; q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : 🔒_\mu \vdash T \;\mathsf{type} \;@\; p}{\Gamma, x : \{\mathfrak{m} : 🔒_\mu\} \rhd T \;\mathsf{ctx} \;@\; q}$$

$$\frac{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{j} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t[🔑_\varepsilon()/\mathfrak{j}] : T[🔑_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then 🔒_κ is **pointed** by $\text{🔑}_\varepsilon : \mathrm{Id} \to \text{🔒}_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t : T}{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon] : T[\text{🔑}_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔑}_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔑}_\varepsilon, \text{🔒}_\kappa)$ or
- $(\mathrm{id}_\Gamma, \text{🔑}_\varepsilon, \text{🔒}_\kappa, \text{🔒}_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} \mathrel{@} q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : \text{🔒}_\mu\ \mathsf{ctx} \mathrel{@} p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx} \mathrel{@} q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T\ \mathsf{type} \mathrel{@} p}{\Gamma, x : \{\mathfrak{m} : \text{🔒}_\mu\} \triangleright T\ \mathsf{ctx} \mathrel{@} q}$$

$$\frac{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{j} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon()/\mathfrak{j}] : T[\text{🔑}_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then $🔒_\kappa$ is **pointed** by $🔑_\varepsilon : \mathrm{Id} \to 🔒_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, 🔒_\kappa, 🔒_\kappa, 🔒_\kappa \vdash t : T}{\Gamma, 🔒_\kappa, 🔒_\kappa \vdash t[🔑_\varepsilon] : T[🔑_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔒_\kappa, 🔑_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔑_\varepsilon, 🔒_\kappa)$ or
- $(\mathrm{id}_\Gamma, 🔑_\varepsilon, 🔒_\kappa, 🔒_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} \ @\ q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : 🔒_\mu\ \mathsf{ctx}\ @\ p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx}\ @\ q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : 🔒_\mu \vdash T\ \mathsf{type}\ @\ p}{\Gamma, x : \{\mathfrak{m} : 🔒_\mu\} \rhd T\ \mathsf{ctx}\ @\ q}$$

$$\frac{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{j} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t[🔑_\varepsilon()/\mathfrak{j}] : T[🔑_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then 🔒_κ is **pointed** by $\text{🔑}_\varepsilon : \mathrm{Id} \to \text{🔒}_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t : T}{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon] : T[\text{🔑}_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔑}_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔑}_\varepsilon, \text{🔒}_\kappa)$ or
- $(\mathrm{id}_\Gamma, \text{🔑}_\varepsilon, \text{🔒}_\kappa, \text{🔒}_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : \text{🔒}_\mu \ \mathsf{ctx} \ @ \ p}$$

MTT: Modal variables

$$\frac{\Gamma \ \mathsf{ctx} \ @ \ q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T \ \mathsf{type} \ @ \ p}{\Gamma, x : \{\mathfrak{m} : \text{🔒}_\mu\} \rhd T \ \mathsf{ctx} \ @ \ q}$$

$$\frac{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{j} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon()/\mathfrak{j}] : T[\text{🔑}_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then 🔒_κ is **pointed** by $\text{🔑}_\varepsilon : \mathrm{Id} \to \text{🔒}_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t : T}{\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon] : T[\text{🔑}_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔒}_\kappa, \text{🔑}_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, \text{🔒}_\kappa, \text{🔑}_\varepsilon, \text{🔒}_\kappa)$ or
- $(\mathrm{id}_\Gamma, \text{🔑}_\varepsilon, \text{🔒}_\kappa, \text{🔒}_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : \text{🔒}_\mu\ \mathsf{ctx} @ p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T\ \mathsf{type} @ p}{\Gamma, x : \{\mathfrak{m} : \text{🔒}_\mu\} \triangleright T\ \mathsf{ctx} @ q}$$

$$\frac{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{j} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : \text{🔒}_\kappa, \mathfrak{k} : \text{🔒}_\kappa \vdash t[\text{🔑}_\varepsilon()/\mathfrak{j}] : T[\text{🔑}_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then $🔒_\kappa$ is **pointed** by $🔑_\varepsilon : \mathrm{Id} \to 🔒_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, 🔒_\kappa, 🔒_\kappa, 🔒_\kappa \vdash t : T}{\Gamma, 🔒_\kappa, 🔒_\kappa \vdash t[🔑_\varepsilon] : T[🔑_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔒_\kappa, 🔑_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, 🔒_\kappa, 🔑_\varepsilon, 🔒_\kappa)$ or
- $(\mathrm{id}_\Gamma, 🔑_\varepsilon, 🔒_\kappa, 🔒_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : 🔒_\mu\ \mathsf{ctx} @ p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx} @ q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : 🔒_\mu \vdash T\ \mathsf{type} @ p}{\Gamma, x : \{\mathfrak{m} : 🔒_\mu\} \rhd T\ \mathsf{ctx} @ q}$$

$$\frac{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{j} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : 🔒_\kappa, \mathfrak{k} : 🔒_\kappa \vdash t[🔑_\varepsilon()/\mathfrak{j}] : T[🔑_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Case study: Readability

Let $\kappa : p \to p$ be **copointed** by $\varepsilon : \kappa \Rightarrow \mathrm{id}$.
Then $\lock_\kappa$ is **pointed** by $\key_\varepsilon : \mathrm{Id} \to \lock_\kappa$.

The following is ambiguous:

$$\frac{\Gamma, \lock_\kappa, \lock_\kappa, \lock_\kappa \vdash t : T}{\Gamma, \lock_\kappa, \lock_\kappa \vdash t[\key_\varepsilon] : T[\key_\varepsilon]}$$

Did we use

- $(\mathrm{id}_\Gamma, \lock_\kappa, \lock_\kappa, \key_\varepsilon)$ or
- $(\mathrm{id}_\Gamma, \lock_\kappa, \key_\varepsilon, \lock_\kappa)$ or
- $(\mathrm{id}_\Gamma, \key_\varepsilon, \lock_\kappa, \lock_\kappa)$?

This smells like de Bruijn indices.
Let's use names instead.

MTT: Locks

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} q \qquad \mu : p \to q}{\Gamma, \mathfrak{m} : \lock_\mu\ \mathsf{ctx} \mathbin{@} p}$$

MTT: Modal variables

$$\frac{\Gamma\ \mathsf{ctx} \mathbin{@} q \qquad \mu : p \to q \qquad \Gamma, \mathfrak{m} : \lock_\mu \vdash T\ \mathsf{type} \mathbin{@} p}{\Gamma, x : \{\mathfrak{m} : \lock_\mu\} \triangleright T\ \mathsf{ctx} \mathbin{@} q}$$

$$\frac{\Gamma, \mathfrak{i} : \lock_\kappa, \mathfrak{j} : \lock_\kappa, \mathfrak{k} : \lock_\kappa \vdash t : T}{\Gamma, \mathfrak{i} : \lock_\kappa, \mathfrak{k} : \lock_\kappa \vdash t[\key_\varepsilon()/\mathfrak{j}] : T[\key_\varepsilon()/\mathfrak{j}]}$$

Idea: Substitution replaces **lock variables** with **lock terms**.

Lock Calculus

Ψ ltele @ $q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu$ @ $q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \text{ mode}}{() \text{ ltele @ } p \to p} \qquad \frac{\Psi \text{ ltele @ } r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \text{ ltele @ } r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \text{ @ } q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \text{ @ } q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \text{ @ } r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \text{ @ } q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \text{ @ } r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} \text{ @ } p \to p}$$

Lock Calculus

$\Psi \,\mathsf{ltele} \,@\, q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu \,@\, q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi \,@\, q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \,\mathsf{mode}}{() \,\mathsf{ltele} \,@\, p \to p} \qquad \frac{\Psi \,\mathsf{ltele} \,@\, r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \,\mathsf{ltele} \,@\, r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \,@\, q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \,@\, q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \,@\, r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \,@\, q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \,@\, r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} \,@\, p \to p}$$

Lock Calculus

Ψ ltele @ $q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu$ @ $q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \text{ mode}}{() \text{ ltele @ } p \to p} \qquad \frac{\Psi \text{ ltele @ } r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \text{ ltele @ } r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \text{ @ } q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \text{ @ } q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \text{ @ } r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \text{ @ } q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \text{ @ } r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} \text{ @ } p \to p}$$

Lock Calculus

Ψ ltele @ $q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu$ @ $q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \text{ mode}}{() \text{ ltele @ } p \to p} \qquad \frac{\Psi \text{ ltele @ } r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \text{ ltele @ } r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \text{ @ } q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \text{ @ } q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \text{ @ } r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \text{ @ } q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \text{ @ } r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} \text{ @ } p \to p}$$

Lock Calculus

$\Psi\ \text{ltele} @ q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu @ q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi @ q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p\ \text{mode}}{()\ \text{ltele} @ p \to p} \qquad \frac{\Psi\ \text{ltele} @ r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu\ \text{ltele} @ r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu @ q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu @ q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu @ r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu @ q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} @ r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} @ p \to p}$$

Lock Calculus

Ψ ltele @ $q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu$ @ $q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \text{ mode}}{() \text{ ltele @ } p \to p} \qquad \frac{\Psi \text{ ltele @ } r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \text{ ltele @ } r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \text{ @ } q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \text{ @ } q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \text{ @ } r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \text{ @ } q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \text{ @ } r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}} \text{ @ } p \to p}$$

Lock Calculus

$\Psi\ \text{ltele} \ @\ q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu \ @\ q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi \ @\ q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p\ \text{mode}}{()\ \text{ltele} \ @\ p \to p} \qquad \frac{\Psi\ \text{ltele} \ @\ r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu\ \text{ltele} \ @\ r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \ @\ q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \ @\ q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \ @\ r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \ @\ q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \ @\ r \to p} \qquad \frac{}{() \vdash () : 🔒_{\text{id}} \ @\ p \to p}$$

Lock Calculus

$\Psi \,\text{ltele} \,@\, q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu \,@\, q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi \,@\, q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p \,\text{mode}}{() \,\text{ltele} \,@\, p \to p} \qquad \frac{\Psi \,\text{ltele} \,@\, r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu \,\text{ltele} \,@\, r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu \,@\, q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu \,@\, q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu \,@\, r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu \,@\, q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu} \,@\, r \to p} \qquad \frac{}{() \vdash () : 🔒_{\text{id}} \,@\, p \to p}$$

Lock Calculus

$\Psi\ \mathsf{ltele}\ @\ q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu\ @\ q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi\ @\ q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock telescopes:

$$\frac{p\ \mathsf{mode}}{()\ \mathsf{ltele}\ @\ p \to p} \qquad \frac{\Psi\ \mathsf{ltele}\ @\ r \to q \qquad \mu : p \to q}{\Psi, \mathfrak{m} : 🔒_\mu\ \mathsf{ltele}\ @\ r \to p}$$

Lock terms (**no** weakening, **no** exchange, **no** contraction):

$$\frac{\mu : p \to q}{\mathfrak{m} : 🔒_\mu \vdash \mathfrak{m} : 🔒_\mu\ @\ q \to p} \qquad \frac{\Psi \vdash \mathfrak{t} : 🔒_\nu \qquad \alpha : \mu \Rightarrow \nu : p \to q}{\Psi \vdash 🔑_\alpha(\mathfrak{t}) : 🔒_\mu\ @\ q \to p}$$

$$\frac{\Phi \vdash \mathfrak{s} : 🔒_\nu\ @\ r \to q \qquad \Psi \vdash \mathfrak{t} : 🔒_\mu\ @\ q \to p}{\Phi, \Psi \vdash (\mathfrak{s}, \mathfrak{t}) : 🔒_{\nu \circ \mu}\ @\ r \to p} \qquad \frac{}{() \vdash () : 🔒_{\mathrm{id}}\ @\ p \to p}$$

Ψ ltele $@\, q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_{\mu} \,@\, q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_{\mu}]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi \,@\, q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock substitutions:

- Arise from terms: $$\frac{\Psi \vdash \mathfrak{t} : 🔒_{\mu} \,@\, q \to p}{\Psi \vdash (\mathfrak{t}/\mathfrak{m}) : (\mathfrak{m} : 🔒_{\mu}) \,@\, q \to p}$$
- Compose horizontally: $$\frac{\Psi' \vdash \mathfrak{S} : \Phi' \,@\, r \to q \qquad \Psi \vdash \mathfrak{T} : \Phi \,@\, q \to p}{\Psi', \Psi \vdash \mathfrak{S}, \mathfrak{T} : \Phi', \Phi \,@\, r \to p}$$
- Compose vertically (by substitution): $$\frac{\Psi \vdash \mathfrak{T} : \Xi \,@\, q \to p \qquad \Phi \vdash \mathfrak{S} : \Psi \,@\, q \to p}{\Phi \vdash \mathfrak{T}[\mathfrak{S}] : \Xi \,@\, q \to p}$$

$\Psi\ \mathsf{ltele} \ @\ q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu \ @\ q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi \ @\ q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock substitutions:

- Arise from terms: $$\frac{\Psi \vdash \mathfrak{t} : 🔒_\mu \ @\ q \to p}{\Psi \vdash (\mathfrak{t}/\mathfrak{m}) : (\mathfrak{m} : 🔒_\mu) \ @\ q \to p}$$
- Compose horizontally: $$\frac{\Psi' \vdash \mathfrak{S} : \Phi' \ @\ r \to q \qquad \Psi \vdash \mathfrak{T} : \Phi \ @\ q \to p}{\Psi', \Psi \vdash \mathfrak{S}, \mathfrak{T} : \Phi', \Phi \ @\ r \to p}$$
- Compose vertically (by substitution): $$\frac{\Psi \vdash \mathfrak{T} : \Xi \ @\ q \to p \qquad \Phi \vdash \mathfrak{S} : \Psi \ @\ q \to p}{\Phi \vdash \mathfrak{T}[\mathfrak{S}] : \Xi \ @\ q \to p}$$

$\Psi\ \mathsf{ltele}\ @\ q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : \text{🔒}_\mu\ @\ q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![\text{🔒}_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi\ @\ q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock substitutions:

- Arise from terms: $$\frac{\Psi \vdash \mathfrak{t} : \text{🔒}_\mu\ @\ q \to p}{\Psi \vdash (\mathfrak{t}/\mathfrak{m}) : (\mathfrak{m} : \text{🔒}_\mu)\ @\ q \to p}$$
- Compose horizontally: $$\frac{\Psi' \vdash \mathfrak{S} : \Phi'\ @\ r \to q \qquad \Psi \vdash \mathfrak{T} : \Phi\ @\ q \to p}{\Psi', \Psi \vdash \mathfrak{S}, \mathfrak{T} : \Phi', \Phi\ @\ r \to p}$$
- Compose vertically (by substitution): $$\frac{\Psi \vdash \mathfrak{T} : \Xi\ @\ q \to p \qquad \Phi \vdash \mathfrak{S} : \Psi\ @\ q \to p}{\Phi \vdash \mathfrak{T}[\mathfrak{S}] : \Xi\ @\ q \to p}$$

Ψ ltele @ $q \to p$	Ψ is a lock telescope $q \to p$	$[\![\Psi]\!]$ is a functor $[\![q]\!] \to [\![p]\!]$.
$\Psi \vdash \mathfrak{t} : 🔒_\mu$ @ $q \to p$	$\mathfrak{t}$ is a lock term in ctx. Ψ	$[\![\mathfrak{t}]\!] : [\![\Psi]\!] \to [\![🔒_\mu]\!]$ is a nat. transf.
$\Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$	$\mathfrak{T}$ is a lock subst. from Ψ to Φ	$[\![\mathfrak{T}]\!] : [\![\Psi]\!] \to [\![\Phi]\!]$ is a nat. transf.

Lock substitutions:

- Arise from terms: $$\frac{\Psi \vdash \mathfrak{t} : 🔒_\mu \text{ @ } q \to p}{\Psi \vdash (\mathfrak{t}/\mathfrak{m}) : (\mathfrak{m} : 🔒_\mu) \text{ @ } q \to p}$$
- Compose horizontally: $$\frac{\Psi' \vdash \mathfrak{S} : \Phi' \text{ @ } r \to q \qquad \Psi \vdash \mathfrak{T} : \Phi \text{ @ } q \to p}{\Psi', \Psi \vdash \mathfrak{S}, \mathfrak{T} : \Phi', \Phi \text{ @ } r \to p}$$
- Compose vertically (by substitution): $$\frac{\Psi \vdash J \text{ @ } q \to p \qquad \Phi \vdash \mathfrak{S} : \Psi \text{ @ } q \to p}{\Phi \vdash J[\mathfrak{S}] \text{ @ } q \to p}$$

Intermezzo: Modal Type Notation

MTT

$$\frac{\Gamma, 🔒_\mu \vdash T \text{ type}}{\Gamma \vdash \langle \mu \mid T \rangle \text{ type}}$$

$$\frac{\Gamma, 🔒_\mu \vdash t : T}{\Gamma \vdash \text{mod}_\mu\, t : \langle \mu \mid T \rangle}$$

Named MTT

$$\frac{\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \text{ type}}{\Gamma \vdash (\mathfrak{m} : 🔒_\mu) \to T \text{ type}}$$

$$\frac{\Gamma, \mathfrak{m} : 🔒_\mu \vdash t : T}{\Gamma \vdash \lambda(\mathfrak{m} : 🔒_\mu).t : (\mathfrak{m} : 🔒_\mu) \to T}$$

Intermezzo: Modal Type Notation

MTT

$$\frac{\Gamma, \text{🔒}_\mu \vdash T \text{ type}}{\Gamma \vdash \langle \mu \mid T \rangle \text{ type}}$$

$$\frac{\Gamma, \text{🔒}_\mu \vdash t : T}{\Gamma \vdash \text{mod}_\mu\, t : \langle \mu \mid T \rangle}$$

Named MTT

$$\frac{\Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T \text{ type}}{\Gamma \vdash (\mathfrak{m} : \text{🔒}_\mu) \to T \text{ type}}$$

$$\frac{\Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash t : T}{\Gamma \vdash \lambda(\mathfrak{m} : \text{🔒}_\mu).t : (\mathfrak{m} : \text{🔒}_\mu) \to T}$$

The Splitting Problem

MTT:

$$\Gamma, \text{🔒}_{\nu \circ \mu} = \Gamma, \text{🔒}_{\nu}, \text{🔒}_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : \text{🔒}_{\nu \circ \mu} = \Gamma, \mathfrak{n} : \text{🔒}_{\nu}, \mathfrak{m} : \text{🔒}_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[\text{🔑}_{\varepsilon}]$$

$$(\Gamma, \text{🔒}_{\kappa}, \text{🔒}_{\mu}) = (\Gamma, \text{🔒}_{\kappa \circ \mu}) \xleftarrow{\text{🔑}_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : \text{🔒}_{\kappa}\} \triangleright (\mathfrak{m} : \text{🔒}_{\mu}) \to T) \to T[\text{🔑}_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

MTT:

$$\Gamma, 🔒_{\nu\circ\mu} = \Gamma, 🔒_{\nu}, 🔒_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : 🔒_{\nu\circ\mu} = \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[🔑_{\varepsilon}]$$

$$(\Gamma, 🔒_{\kappa}, 🔒_{\mu}) = (\Gamma, 🔒_{\kappa\circ\mu}) \xleftarrow{🔑_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to T[🔑_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

The Splitting Problem

MTT:

$$\Gamma, 🔒_{\nu \circ \mu} \quad = \quad \Gamma, 🔒_{\nu}, 🔒_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : 🔒_{\nu \circ \mu} \quad = \quad \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[🔑_{\varepsilon}]$$

$$(\Gamma, 🔒_{\kappa}, 🔒_{\mu}) = (\Gamma, 🔒_{\kappa \circ \mu}) \xleftarrow{🔑_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to T[🔑_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

The Splitting Problem

MTT:

$$\Gamma, \text{🔒}_{\nu\circ\mu} \quad = \quad \Gamma, \text{🔒}_{\nu}, \text{🔒}_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : \text{🔒}_{\nu\circ\mu} \quad = \quad \Gamma, \mathfrak{n} : \text{🔒}_{\nu}, \mathfrak{m} : \text{🔒}_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[\text{🔑}_{\varepsilon}]$$

$$(\Gamma, \text{🔒}_{\kappa}, \text{🔒}_{\mu}) = (\Gamma, \text{🔒}_{\kappa\circ\mu}) \xleftarrow{\text{🔑}_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : \text{🔒}_{\kappa}\} \triangleright (\mathfrak{m} : \text{🔒}_{\mu}) \to T) \to T[\text{🔑}_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

The Splitting Problem

MTT:

$$\Gamma, 🔒_{\nu\circ\mu} = \Gamma, 🔒_{\nu}, 🔒_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : 🔒_{\nu\circ\mu} = \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[🔑_{\varepsilon}]$$

$$(\Gamma, 🔒_{\kappa}, 🔒_{\mu}) = (\Gamma, 🔒_{\kappa\circ\mu}) \xleftarrow{🔑_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to T[🔑_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

The Splitting Problem

MTT:

$$\Gamma, \text{🔒}_{\nu\circ\mu} \quad = \quad \Gamma, \text{🔒}_{\nu}, \text{🔒}_{\mu}$$

Named MTT:

$$\Gamma, \mathfrak{o} : \text{🔒}_{\nu\circ\mu} \quad = \quad \Gamma, \mathfrak{n} : \text{🔒}_{\nu}, \mathfrak{m} : \text{🔒}_{\mu}$$

Crucially used e.g. in projection for internal right adjoint modalities.

Internal adjunction

Assume $\kappa \dashv \mu$ as witnessed by $\eta : \mathrm{id} \Rightarrow \mu \circ \kappa$ and $\varepsilon : \kappa \circ \mu \Rightarrow \mathrm{id}$

MTT: Projection

$$(\kappa \mid \langle \mu \mid T \rangle) \to T[\text{🔑}_{\varepsilon}]$$

$$(\Gamma, \text{🔒}_{\kappa}, \text{🔒}_{\mu}) = (\Gamma, \text{🔒}_{\kappa\circ\mu}) \xleftarrow{\text{🔑}_{\varepsilon}} \Gamma$$

Named MTT: Projection

$$(\{\mathfrak{k} : \text{🔒}_{\kappa}\} \triangleright (\mathfrak{m} : \text{🔒}_{\mu}) \to T) \to T[\text{🔑}_{\varepsilon}()/\mathfrak{o}]$$

Cannot be done syntactically!

Looking for inspiration...

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?
They have **eliminators:**

- $\mathsf{let}\,((\mathfrak{n},\mathfrak{m}) = \mathfrak{t})\,\mathsf{in}\ldots$
- $\mathsf{let}\,(() = \mathfrak{t})\,\mathsf{in}\ldots$ (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : 🔒_{\nu\circ\mu} \quad \overset{\neq}{\approx} \quad \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Nice: Many models don't have $=$
$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to$$
$$\mathsf{let}\,((\mathfrak{k},\mathfrak{m}) = 🔑_{\varepsilon}())\,\mathsf{in}\,T$$

Looking for inspiration...

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?
They have **eliminators:**

- let $((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})$ in ...
- let $(() = \mathfrak{t})$ in ... (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : 🔒_{\nu \circ \mu} \not\approx \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Nice: Many models don't have =
$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to \text{let}\,((\mathfrak{k}, \mathfrak{m}) = 🔑_{\varepsilon}())\,\text{in}\,T$$

Looking for inspiration...

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?
They have **eliminators:**

- let $((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})$ in ...
- let $(() = \mathfrak{t})$ in ... (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : \text{🔒}_{\nu \circ \mu} \overset{\neq}{\approx} \Gamma, \mathfrak{n} : \text{🔒}_{\nu}, \mathfrak{m} : \text{🔒}_{\mu}$$

Nice: Many models don't have $=$
$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : \text{🔒}_{\kappa}\} \triangleright (\mathfrak{m} : \text{🔒}_{\mu}) \to T) \to$$
$$\text{let}\,((\mathfrak{k}, \mathfrak{m}) = \text{🔑}_{\varepsilon}())\,\text{in}\,T$$

Looking for inspiration…

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?

They have **eliminators:**

- $\text{let}\,((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})\,\text{in}\,\ldots$
- $\text{let}\,(() = \mathfrak{t})\,\text{in}\,\ldots$ (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : \text{🔒}_{\nu\circ\mu} \quad \not\approx \quad \Gamma, \mathfrak{n} : \text{🔒}_{\nu}, \mathfrak{m} : \text{🔒}_{\mu}$$

Nice: Many models don't have $=$

$\leadsto$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : \text{🔒}_{\kappa}\} \triangleright (\mathfrak{m} : \text{🔒}_{\mu}) \to T) \to \text{let}\,((\mathfrak{k}, \mathfrak{m}) = \text{🔑}_{\varepsilon}())\,\text{in}\,T$$

Looking for inspiration. . .

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?
They have **eliminators:**

- $\mathrm{let}\,((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})\,\mathrm{in}\,\ldots$
- $\mathrm{let}\,(() = \mathfrak{t})\,\mathrm{in}\,\ldots$ (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : 🔒_{\nu \circ \mu} \quad \not\approx \quad \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Nice: Many models don't have $=$
$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to$$
$$\mathrm{let}\,((\mathfrak{k}, \mathfrak{m}) = 🔑_{\varepsilon}())\,\mathrm{in}\,T$$

Looking for inspiration...

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?
They have **eliminators:**

- let $((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})$ in ...
- let $(() = \mathfrak{t})$ in ... (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : 🔒_{\nu \circ \mu} \quad \overset{\neq}{\approx} \quad \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Nice: Many models don't have $=$
$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to$$
$$\text{let}\,((\mathfrak{k}, \mathfrak{m}) = 🔑_{\varepsilon}())\,\text{in}\,T$$

Looking for inspiration...

- Lock calculus is the internal language of the **mode theory**.
- Mode theory is an arbitrary **2-category**.
- Single-object 2-category is a **monoidal category**.

What do internal languages for monoidal categories do?

They have **eliminators:**

- $\mathsf{let}\,((\mathfrak{n}, \mathfrak{m}) = \mathfrak{t})\,\mathsf{in}\,\ldots$
- $\mathsf{let}\,(() = \mathfrak{t})\,\mathsf{in}\,\ldots$ (no weakening!)

(Jaskelioff & Moggi, 2010; Shulman, 2016)

$$\Gamma, \mathfrak{o} : 🔒_{\nu \circ \mu} \quad \overset{\neq}{\approx} \quad \Gamma, \mathfrak{n} : 🔒_{\nu}, \mathfrak{m} : 🔒_{\mu}$$

Nice: Many models don't have $=$

$\rightsquigarrow$ no longer need to strictify!

MTT: Projection

$$(\{\mathfrak{k} : 🔒_{\kappa}\} \triangleright (\mathfrak{m} : 🔒_{\mu}) \to T) \to$$
$$\mathsf{let}\,((\mathfrak{k}, \mathfrak{m}) = 🔑_{\varepsilon}())\,\mathsf{in}\,T$$

The Metamode

Infinite Mode Theories

Systems with infinite mode theory:

- Degrees of Relatedness ($p \in \mathbb{Z}_{\geq -2}$)
- Transpension System
  (modes are shape contexts)

$\Rightarrow$ Code ∞plication unsustainable,
need internal
mode/modality/2-cell polymorphism.

Degrees of Relatedness

$\mathbf{par} : (p : \mathrm{Mode}) \to \mathrm{Modty}(p+1, p)$
$\mathrm{id} : (p : \mathrm{Mode}) \to (\mathbf{par}\, p \mid X : \mathscr{U}_\ell^{p}) \to X \to X$

Infinite Mode Theories

Systems with infinite mode theory:

- Degrees of Relatedness ($p \in \mathbb{Z}_{\geq -2}$)
- Transpension System
  (modes are shape contexts)

$\Rightarrow$ Code ∞plication unsustainable,
need internal
mode/modality/2-cell polymorphism.

Degrees of Relatedness

$\mathbf{par} : (p : \mathrm{Mode}) \to \mathrm{Modty}(p+1, p)$
$\mathrm{id} : (p : \mathrm{Mode}) \to (\mathbf{par}\, p \mid X : \mathscr{U}_\ell^p) \to X \to X$

Infinite Mode Theories

Systems with infinite mode theory:

- Degrees of Relatedness ($p \in \mathbb{Z}_{\geq -2}$)
- Transpension System
  (modes are shape contexts)

$\Rightarrow$ Code ∞plication unsustainable,
need internal
mode/modality/2-cell polymorphism.

Degrees of Relatedness

$\textbf{par} : (p : \mathsf{Mode}) \to \mathsf{Modty}(p+1, p)$
$\mathsf{id} : (p : \mathsf{Mode}) \to (\textbf{par}\, p \mid X : \mathcal{U}_\ell^p) \to X \to X$

Infinite Mode Theories

Systems with infinite mode theory:

- Degrees of Relatedness ($p \in \mathbb{Z}_{\geq -2}$)
- Transpension System
  (modes are shape contexts)

$\Rightarrow$ Code ∞plication unsustainable,
need internal
mode/modality/2-cell polymorphism.

Degrees of Relatedness

$\textbf{par} : (p : \text{Mode}) \to \text{Modty}(p+1, p)$
$\text{id} : (p : \text{Mode}) \to (\textbf{par}\, p \mid X : \mathscr{U}_\ell^p) \to X \to X$

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T \,\text{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : \text{🔒}_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}]\,\text{type}$

When are these types convertible?
$\leadsto$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly ∘ forever)
  - Time warps
- Degrees of Relatedness (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories non-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \text{ type} @ p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu @ q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \text{ type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
(later, always = constantly ∘ forever)
  - Time warps
- Degrees of Relatedness
(parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \,\text{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly $\circ$ forever)
  - Time warps
- Degrees of Relatedness (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T\,\text{type} \mathbin{@} p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \mathbin{@} q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}]\,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly ∘ forever)
  - Time warps
- Degrees of Relatedness (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories non-freely extended using internal **mode/modality/2-cell polymorphism**

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \text{ type} @ p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu @ q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \text{ type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
  (later, always = constantly ∘ forever)
  - Time warps (Guatto 2018)
- Degrees of Relatedness
  (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T\,\text{type} \mathbin{@} p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : \text{🔒}_\mu \mathbin{@} q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}]\,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly ∘ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_{\mu} \vdash T \,\mathsf{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_{\mu} \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\mathsf{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly ∘ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : \text{🔒}_\mu \vdash T \,\mathsf{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : \text{🔒}_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\mathsf{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
  (later, always = constantly $\circ$ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness
  (parametricity, irrelevance, algebra, . . .)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non**-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \,\text{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
  (later, always = constantly ∘ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness
  (parametricity, irrelevance, algebra, ...)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories non-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \,\mathsf{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\mathsf{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
  (later, always = constantly ∘ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness
  (parametricity, irrelevance, algebra, . . .)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories non-freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T\,\text{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}]\,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory
  (later, always = constantly ∘ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness
  (parametricity, irrelevance, algebra, . . .)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non-**freely extended using internal **mode/modality/2-cell polymorphism**

Lock Term Equality

Assume

- $\Gamma, \mathfrak{m} : 🔒_\mu \vdash T \,\text{type} \,@\, p$
- $\Psi \vdash \mathfrak{s}, \mathfrak{t} : 🔒_\mu \,@\, q \to p$

Then $\Gamma, \Psi \vdash T[\mathfrak{s}/\mathfrak{m}], T[\mathfrak{t}/\mathfrak{m}] \,\text{type}$

When are these types convertible?
$\rightsquigarrow$ When are lock terms equal?

Need **internal** reasoning about **lock term equality**

2-posetal mode theories: Always

- Guarded type theory (later, always = constantly $\circ$ forever)
  - Time warps (Guatto, 2018)
- Degrees of Relatedness (parametricity, irrelevance, algebra, . . .)

(Practically) undecidable

- Modal Transpension System (MTraS)
- Mode theories **non-**freely extended using internal **mode/modality/2-cell polymorphism**

The Metamode

…is not a mode but a metamode:

- Full copy of DTT,
  - Θ ctx
  - $\Theta \vdash T$ type
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi$ ltele $@\, q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{t} : 🔒_{\mu} \;@\, q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi \;@\, q \to p$
  - $\Theta \vdash \Gamma$ ctx $@\, p$
  - $\Theta \vdash \Gamma \vdash T$ type $@\, p$
  - $\Theta \vdash \Gamma \vdash t : T \;@\, p$

- Metamode has types

$$\text{Mode}, \quad \text{Modty}(p, q), \quad \text{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types, for use with metamode's J-rule.

The Metamode

…is not a mode but a metamode:

- Full copy of DTT,
  - Θ ctx
  - $\Theta \vdash T$ type
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi$ ltele @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathtt{t} : 🔒_{\mu}$ @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$
  - $\Theta \vdash \Gamma$ ctx @ p
  - $\Theta \vdash \Gamma \vdash T$ type @ p
  - $\Theta \vdash \Gamma \vdash t : T$ @ p
- Metamode has types

$$\text{Mode}, \quad \text{Modty}(p, q), \quad \text{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types,
  for use with metamode's J-rule.

The Metamode

...is not a mode but a metamode:

- Full copy of DTT,
  - Θ ctx
  - $\Theta \vdash T$ type
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi$ ltele @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathtt{t} : 🔒_\mu$ @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$
  - $\Theta \vdash \Gamma$ ctx @ p
  - $\Theta \vdash \Gamma \vdash T$ type @ p
  - $\Theta \vdash \Gamma \vdash t : T$ @ p

- Metamode has types

$$\text{Mode}, \quad \text{Modty}(p, q), \quad \text{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types,
  for use with metamode's J-rule.

The Metamode

...is not a mode but a metamode:

- Full copy of DTT,
  - Θ ctx
  - $\Theta \vdash T$ type
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi$ ltele @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{t} : 🔒_{\mu}$ @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$
  - $\Theta \vdash \Gamma$ ctx @ p
  - $\Theta \vdash \Gamma \vdash T$ type @ p
  - $\Theta \vdash \Gamma \vdash t : T$ @ p

- Metamode has types

$$\text{Mode}, \quad \text{Modty}(p, q), \quad \text{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types, for use with metamode's J-rule.

The Metamode

...is not a mode but a metamode:

- Full copy of DTT,
  - $\Theta\ \mathsf{ctx}$
  - $\Theta \vdash T\ \mathsf{type}$
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi\ \mathsf{ltele} \ @\ q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{t} : 🔒_{\mu} \ @\ q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi \ @\ q \to p$
  - $\Theta \vdash \Gamma\ \mathsf{ctx} \ @\ p$
  - $\Theta \vdash \Gamma \vdash T\ \mathsf{type} \ @\ p$
  - $\Theta \vdash \Gamma \vdash t : T \ @\ p$

- Metamode has types

$$\mathsf{Mode}, \quad \mathsf{Modty}(p, q), \quad \mathsf{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types, for use with metamode's J-rule.

The Metamode

…is not a mode but a metamode:

- Full copy of DTT,
  - Θ ctx
  - $\Theta \vdash T$ type
  - $\Theta \vdash t :: T$
- Modelled in Set,
- Prefix every MTT or Lock Calculus (LC) judgement with a metamode context:
  - $\Theta \mid \Psi$ ltele @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{t} : 🔒_{\mu}$ @ $q \to p$
  - $\Theta \mid \Psi \vdash \mathfrak{T} : \Phi$ @ $q \to p$
  - $\Theta \vdash \Gamma$ ctx @ p
  - $\Theta \vdash \Gamma \vdash T$ type @ p
  - $\Theta \vdash \Gamma \vdash t : T$ @ p

- Metamode has types

$$\text{Mode}, \quad \text{Modty}(p, q), \quad \text{2Cell}(\mu, \nu)$$

- MTT/LC judgements reflected as metamode types, for use with metamode's J-rule.

Conclusion

By syntactifying the mode theory, we can:

- Make substitution a **replacement** operation again,
- Omit modal **strictification**,
- Have internal mode/modality/2-cell **polymorphism**,
- Give up decidability of **2-cell equality** without problems.

Thanks!

Questions?

Conclusion

By syntactifying the mode theory, we can:

- Make substitution a **replacement** operation again,
- Omit modal **strictification**,
- Have internal mode/modality/2-cell **polymorphism**,
- Give up decidability of **2-cell equality** without problems.

Thanks!

Questions?

Conclusion

By syntactifying the mode theory, we can:

- Make substitution a **replacement** operation again,
- Omit modal **strictification**,
- Have internal mode/modality/2-cell **polymorphism**,
- Give up decidability of **2-cell equality** without problems.

Thanks!

Questions?

Conclusion

By syntactifying the mode theory, we can:

- Make substitution a **replacement** operation again,
- Omit modal **strictification**,
- Have internal mode/modality/2-cell **polymorphism**,
- Give up decidability of **2-cell equality** without problems.

Thanks!

Questions?

Conclusion

By syntactifying the mode theory, we can:

- Make substitution a **replacement** operation again,
- Omit modal **strictification**,
- Have internal mode/modality/2-cell **polymorphism**,
- Give up decidability of **2-cell equality** without problems.

Thanks!

Questions?